media unei variabile aliatoare = expectation for a random variable ( E X = LINIE \* COLOANA )

dispersia unei variabile aliatoare = variance for a random variable ( VAR[X] = E [X2] – ( E[X] )2 )

distributia unei variabile aliatoare = distribution of a random variable ( TABEL)

I. 1.

E X = (-1) \* ¼ + 1 \* ¼ + 2 \* 3/16 + 3 \* 5/16 = 21/16

E Y = 0 \* 1/3 + 1 \* 5/9 + 2 \* 1/9 = 7/9

E Z = (-2) \* 1/7 + 0 \* 4/7 + 2 \* 2/7 = 2/7

E X2 = 1 \* ¼ + 1 \* ¼ + 4 \* 3/16 + 9 \* 5/16 = 65/16

E Y2 = 0 \* 1/3 + 1 \* 5/9 + 4 \* 1/9 = 9/9

E Z2 = 4 \* 1/7 + 0 \* 4/7 + 4 \* 2/7 = 12/7

Var [X] = 65/16 – 441/256 = 599/256

Var [Y] = 1 – 49/81 = 32/81

Var [Z] = 12/7 – 4/49 = 80/49

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X: | -1 | 1 | 2 | 3 |
| 1/4 | 1/4 | 3/16 | 5/16 |

|  |  |  |  |
| --- | --- | --- | --- |
| Y: | 0 | 1 | 2 |
| 1/3 | 5/9 | 1/9 |

|  |  |  |  |
| --- | --- | --- | --- |
| Z: | -2 | 0 | 2 |
| 1/7 | 4/7 | 2/7 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X2: | 1 | 1 | 4 | 9 |
| 1/4 | 1/4 | 3/16 | 5/16 |

|  |  |  |  |
| --- | --- | --- | --- |
| Y2: | 0 | 1 | 4 |
| 1/3 | 5/9 | 1/9 |

|  |  |  |  |
| --- | --- | --- | --- |
| Z2: | 4 | 0 | 4 |
| 1/7 | 4/7 | 2/7 |

I. 4.

{t,t,t} = 0-3 = -3

{h,t,t} = {t,h,t} = {t,t,h} = 1-2 = -1

{h,h,t} = {t,h,h} = {h,t,h} = 2-1 = 1

{h,h,h} = 3-0 = 3

E X = -3 \* 1/8 + -1 \* 3/8 + 1 \* 3/8 + 3 \* 1/8 = 0

E X2 = 9 \* 1/8 + 1 \* 3/8 + 1 \* 3/8 + 9 \* 1/8 = 24/8

Var [X] = 24/8 – 0 = 3

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| --- | --- | --- | --- | --- |
| X: | -3 | -1 | 1 | 3 |
| 1/8 | 3/8 | 3/8 | 1/8 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X2: | 9 | 1 | 1 | 9 |
| 1/8 | 3/8 | 3/8 | 1/8 |

I. 7.

P (X=1) = {1,1}{1,2}{1,3}{1,4}{1,5}{1,6}{2,1}{3,1}{4,1}{5,1}{6,1}

P (X=2) = {2,2}{2,3}{2,4}{2,5}{2,6}{3,2}{4,2}{5,2}{6,2}

P (X=3) = {3,3}{3,4}{3,5}{3,6}{4,3}{5,3}{6,3}

P (X=4) = {4,4}{4,5}{4,6}{5,4}{6,4}

P (X=5) = {5,5}{5,6}{6,5}

P (X=6) = {6,6}

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X: | 1 | 2 | 3 | 4 | 5 | 6 |
| 11/36 | 9/36 | 7/36 | 5/36 | 3/36 | 1/36 |

I. 8.

U1 = 1w, 1b; U2 = 2w, 6b; U3 = 1w, 3b;

X = nr of w



E X = 0 \* 28/90 + 1 \* 37/90 + 2 \* 19/90 + 3 \* 6/90 = 93/90

E X2 = 0 \* 28/90 + 1 \* 37/90 + 4 \* 19/90 + 9 \* 6/90 = 167/90

Var [X] = 167/90 – 8649/8100 = 6381/8100

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X: | 0 | 1 | 2 | 3 |
| 28/90 | 37/90 | 19/90 | 6/90 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X2: | 0 | 1 | 4 | 9 |
| 28/90 | 37/90 | 19/90 | 6/90 |

I. 9.

X = nr of girls

{b, fb, ffb, fff}

P (X=0) = 1/2

P (X=1) = 1/2 \* 1/2 = 1/4

P (X=2) = 1/2 \* 1/2 \* 1/2 = 1/8

P (X=3) = 1/2 \* 1/2 \* 1/2 = 1/8

E X = 0 \* 4/8 + 1 \* 2/8 + 2 \* 1/8 + 3 \* 1/8 = 7/8

E X2 = 0 \* 4/8 + 1 \* 2/8 + 4 \* 1/8 + 9 \* 1/8 = 15/8

Var [X] = 15/8 - 49/64 = 71/64

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X: | 0 | 1 | 2 | 3 |
| 4/8 | 2/8 | 1/8 | 1/8 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X2: | 0 | 1 | 4 | 9 |
| 4/8 | 2/8 | 1/8 | 1/8 |

I. 15.

P1, P2 ; 2 sets won by one of the players P (P1) = 1/3;

X = nr of sets played by P1; Y = nr of sets P2 wins

P (X=2) = 1/9 + 4/9 = 5/9

{P1, P1}= 1/3 \* 1/3 = 1/9

{P2, P2}= 2/3 \* 2/3 = 4/9

P (X=3) = 2/27 + 2/27 + 4/27 + 4/27 = 12/27 = 4/9

{P1, P2, P1}= {P2, P1, P1}= 1/3 \* 2/3 \* 1/3 = 2/27

{P2, P1, P2}= {P1, P2, P2}= 2/3 \* 1/3 \* 2/3 = 4/27

P (Y=0) = {P1, P1}= 1/3 \* 1/3 = 1/9 = 3/27

P (Y=1) = {P1, P2, P1}+ {P2, P1, P1}= 2/27 + 2/27 = 4/27

P (Y=2) = {P2, P1, P2}+{P1, P2, P2}+{P2, P2}= 4/27 + 4/27 + 4/9 = 20/27

|  |  |  |
| --- | --- | --- |
| X: | 2 | 3 |
| 5/9 | 4/9 |

|  |  |  |  |
| --- | --- | --- | --- |
| Y: | 0 | 1 | 2 |
| 3/27 | 4/27 | 20/27 |

X ~ B(n,p) : X = “nr de succese”

n = nr de repetari

E [X] = n \* p p = probabilitate de succese

Var [X] = n \* p \* (1 – p)

II. 1.

2 coins flipped 7 times each; P (h,h)

P (h,h) = 1/2 \* 1/2 = 1/4

X~B (7,1/4)

E X = 7 \* 1/4 = 7/4

Var [X] = 7 \* 1/4 \* 3/4 = 21/16

II. 2.

P (0) = 6/10; P (1) = 4/10;

1. {x,x,x,x,x,x,x} P of five 0’s and two 1’s

(6/10)5 \* (4/10)2 \* C57

1. X = “nr of 0”

X~B(5,6/10)

E X = 5 \* 3/5 = 3

II. 6.

10 cards (out of 52); X = nr of clubs

P (club) = 13/52

X ~ B (10, 1/4)

E X = 10 \* 1/4 = 5/2

X~G (p); X = “nr experientei la care obtinem 1-a oara succes”

E X = 1/p

Var [X] = 1-p/p2

II. 9.

2 dice, 8 times each; X = even product

P (even product) = 1-P (odd product) = 1 – {1,1; 1,3; 1,5; 3,1; 3,3; 3,5; 5,1; 5,3; 5,5} = 1 – 9/36 = 27/36

X~B (8,27/36)

E X = 8 \* 27/36 = 6

III. 1.

nr of rolls of 2 dice to get a product < 7

{1,1; 1,2; 1,3; 1,4; 1,5; 1,6; 2,1; 2,2; 2,3; 3,1; 3,2; 4,1; 5,1; 6,1} = 14/36

X~G (14/36)

E X = 36/14

III. 4.

nr of withdraws from a deck to get a carp which is not diamond

P (not diamond) = 3/4

X~G (3/4)

E X = 4/3

III. 6.

P (0) = 4/10; pairs of bits; nr of pairs until we get 0-0

P (1) = 6/10

P (0-0) = 4/10 \* 4/10 = 16/100

X~G (16/100)

E X = 100/16

Var [X] = …

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| X: | 1 | 2 | 3 | … | n | … |
| … | … | … | … | p \* (1-p)n-1 | … |

X = nr of accedents between 7-8

X~Poisson (lam)

P (X=n) = e-l \* ln/n!

E X = lam

1. P (X>=3) = 1 – P (X=0) – P(X=2) = 1-e-0.7 \* (1 + 0.7 + 0.49/2 )
2. P (X>=1) = 1 – P (X=0) = 1-e-0.7